

A2T Midterm Review Solutions

1A

$$8^x = 2^{x+6}$$

$$(2^3)^x = 2^{x+6}$$

$$2^{3x} = 2^{x+6}$$

$$3x = x+6$$

$$2x = 6$$

$$\boxed{x = 3}$$

1B

$$4^{2x-3} = \frac{1}{16}$$

$$4^{2x-3} = 4^{-2}$$

$$2x-3 = -2$$

$$2x = 1$$

$$\boxed{x = 1/2}$$

1C

$$(x^{3/2})^{2/3} = (64)^{2/3}$$

$$\boxed{x = 16}$$

1D

$$125^{-2x} = 25^{x+1}$$

$$(5^3)^{-2x} = (5^2)^{x+1}$$

$$5^{-6x} = 5^{2x+2}$$

$$-6x = 2x+2$$

$$-8x = 2$$

$$\boxed{x = -1/4}$$

1E

$$a^{3/5} = 25$$

$$(a^{3/5})^{5/3} = (25)^{5/3}$$

$$\boxed{a = 243}$$

1

$$(1F) \quad 3(2m+3)^{2/3} + 2 = 77$$

$$3(2m+3)^{2/3} = 75$$

$$(2m+3)^{4/3} = 25$$

$$\left((2m+3)^{2/3}\right)^2 = (25)^{3/2}$$

$$2m+3 = 125$$

$$2m = 122$$

$$\boxed{m = 61}$$

$$\text{OR } 2m+3 = -125$$

$$2m = -128$$

$$\boxed{m = -64}$$

Please note:
In a prob
like #1F we
usually
ask for the
principle root
to be used so
only $m = 61$ would
be needed.

$$(2) \quad \frac{6x(x+2)}{x(x^2-5x-14)} = \frac{6\cancel{(x+2)}}{x(x-7)\cancel{(x+2)}}$$

$$= \boxed{\frac{6}{x(x-7)}}$$

$$\textcircled{3} \frac{(x-6)(\cancel{x+4})}{(\cancel{x+4})(x-4)} = \boxed{\frac{x-6}{x-4}}$$

$$\textcircled{4A} \boxed{x=2}$$

$$\textcircled{4B} \boxed{x=0}$$

$$\textcircled{4C} \begin{aligned} x^2 + 6x + 8 &= 0 \\ (x+4)(x+2) &= 0 \\ \boxed{x=-4, x=-2} \end{aligned}$$

$$\textcircled{5} \frac{3}{x+4} + \frac{2}{x}$$

$$= \frac{3x}{x(x+4)} + \frac{2(x+4)}{x(x+4)}$$

$$= \frac{3x + 2x + 8}{x(x+4)} = \boxed{\frac{5x+8}{x(x+4)}}$$

$\textcircled{6}$ Please note: The last fraction has been flipped to the last operation changed to multiplication already

$$\frac{5(x^2 - x - 12)}{x(4x-1)} \cdot \frac{(x-5)(x+2)}{x-4} \cdot \frac{2x(4x-1)}{(x-5)(x+3)}$$

$$\frac{5(\cancel{x-4})(\cancel{x+3})}{\cancel{x(4x-1)}} \cdot \frac{(\cancel{x-5})(x+2)}{\cancel{x-4}} \cdot \frac{2\cancel{x}(4\cancel{x-1})}{(\cancel{x-5})(\cancel{x+3})}$$

$$= \boxed{10(x+2) \text{ or } 10x+20}$$

$\textcircled{3}$

$$\begin{aligned}
 \textcircled{7} \quad & \frac{2x^2-16}{(x-2)(x+2)} - \frac{x+4}{x+2} \cdot \frac{x-2}{x-2} \\
 &= \frac{2x^2-16}{(x-2)(x+2)} - \frac{(x+4)(x-2)}{(x+2)(x-2)} \\
 &= \frac{2x^2-16}{(x-2)(x+2)} - \frac{x^2+2x-8}{(x+2)(x-2)} \\
 &= \frac{x^2-2x-8}{(x-2)(x+2)} = \frac{(x-4)(x+2)}{(x-2)(x+2)} \\
 &= \boxed{\frac{x-4}{x-2}}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{8} \quad & \frac{x-1}{(x+1)(x+2)} + \frac{x}{x+1} \\
 &= \frac{x-1}{(x+1)(x+2)} + \frac{x(x+2)}{(x+1)(x+2)} \\
 &= \frac{x-1+x^2+2x}{(x+1)(x+2)} = \boxed{\frac{x^2+3x-1}{(x+1)(x+2)}}
 \end{aligned}$$

④

$$\textcircled{9} \quad \frac{1}{6x} + \frac{8}{x} = \frac{x}{6}$$

$$6x \left(\frac{1}{6x} + \frac{8}{x} = \frac{x}{6} \right)$$

$$1 + 48 = x^2$$

$$49 = x^2$$

$$\pm 7 = x$$

(10 + 11 are on next pg)

$$\begin{aligned} \textcircled{12} \quad \frac{\sqrt{900}}{\sqrt{20}} &= \frac{30}{\sqrt{4}\sqrt{5}} = \frac{30}{2\sqrt{5}} = \frac{15}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \\ &= \frac{15\sqrt{5}}{5} = \boxed{3\sqrt{5}} \end{aligned}$$

(13 on page 7)

$$\begin{aligned} \textcircled{14} \quad &\sqrt{49a^2b^4} - \sqrt{16a^2b^4} + \sqrt{8a^2c} \\ &= 7ab^2 - 4ab^2 + \sqrt{4}\sqrt{2} \cdot a \cdot \sqrt{c} \\ &= 7ab^2 - 4ab^2 + 2a\sqrt{2c} \\ &= \boxed{3ab^2 + 2a\sqrt{2c}} \end{aligned}$$

⑤

(10)

$$\frac{4n+3}{n-6} + \frac{n-4}{6-n} = \frac{44}{2(n-6)}$$

$$\left(\frac{4n+3}{n-6} + \frac{4-n}{n-6} = \frac{44}{2(n-6)} \right) \cdot 2(n-6)$$

$$2(4n+3) + 2(4-n) = 44$$

$$8n+6+8-2n=44$$

$$6n+14=44$$

$$6n=30$$

$$\boxed{n=5}$$

(11c)

$$\frac{1 - \frac{1}{16x^2}}{1 - \frac{1}{4x}} \cdot \frac{16x^2}{16x^2}$$

$$= \frac{16x^2 - 1}{16x^2 - 4x} = \frac{(4x-1)(4x+1)}{4x(4x-1)}$$

$$\boxed{\frac{4x+1}{4x}}$$

(#11A, B, & D are on p13)

(6)

$$\begin{aligned}
 (13) \quad & 2\sqrt{16}\sqrt{3} + 2\sqrt{4}\sqrt{3} \\
 &= 2 \cdot 4\sqrt{3} + 2 \cdot 2\sqrt{3} \\
 &= 8\sqrt{3} + 4\sqrt{3} = \boxed{12\sqrt{3}}
 \end{aligned}$$

$$(15) \quad \frac{3}{6-5\sqrt{2}} \cdot \frac{6+5\sqrt{2}}{6+5\sqrt{2}}$$

$$= \frac{18 + 15\sqrt{2}}{36 - 25(2)} = \frac{18 + 15\sqrt{2}}{-14}$$

or

$$-\frac{9}{7} - \frac{15}{14}\sqrt{2}$$

(#16 is on p 8)

$$(17) \quad 2\sqrt{2x-6} + 8 = 4$$

$$2\sqrt{2x-6} = -4$$

$$(\sqrt{2x-6})^2 = (-2)^2$$

$$2x - 6 = 4$$

$$2x = 10$$

$x = 5 \leftarrow$ reject

#17 continued

Check:

$$2\sqrt{2(5)-6} + 8 \stackrel{?}{=} 4$$

$$2\sqrt{4} + 8$$

$$2(2) + 8$$

$$4 + 8 \neq 4$$

no solution

$$(16) \sqrt{3x+6} - 2 = 7$$

$$\sqrt{3x+6} = 9$$

$$3x+6 = 81$$

$$3x = 75$$

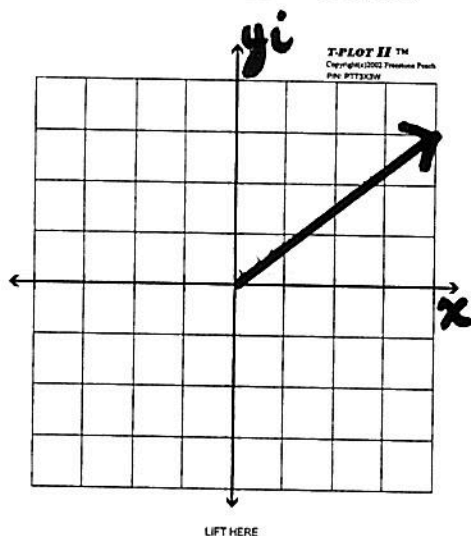
$$x = 25$$

$$(18) \sqrt{-45x^4y^7} = i\sqrt{9}\sqrt{5} \cdot x^2 \cdot y^3\sqrt{y}$$
$$= \boxed{3ix^2y^3\sqrt{5y}}$$

(8)

$$\begin{aligned}
 (19) \quad & 5\sqrt{-18} + \sqrt{50} - \sqrt{-75} \\
 &= 5i\sqrt{9}\sqrt{2} + i\sqrt{5}\sqrt{2} - i\sqrt{5}\sqrt{3} \\
 &= 5i(3)\sqrt{2} + 5i\sqrt{2} - 5i\sqrt{3} \\
 &= 15i\sqrt{2} + 5i\sqrt{2} - 5i\sqrt{3} = 20i\sqrt{2} - 5i\sqrt{3}
 \end{aligned}$$

(20)



(21) i^{53}

$$\frac{53}{4} = 14.25$$

$\underbrace{\hspace{1cm}}$
 we relate
 to i'

so $i^{53} = i' = \boxed{i}$

$$\begin{aligned}
 (22) \quad & (2-5i)(6+7i) \\
 &= 12 + 14i - 30i - 35i^2 \\
 &= 12 - 16i + 35 \\
 &= \boxed{47 - 16i}
 \end{aligned}$$

(23) mult. inv

$$\frac{1}{7-4i} \cdot \frac{7+4i}{7+4i} = \frac{7+4i}{49-16i^2} = \frac{7+4i}{49+16} = \frac{7+4i}{65}$$

or $\frac{7}{65} + \frac{4}{65}i$

magnitude

$$|7+4i| = \sqrt{7^2 + 4^2} = \sqrt{65}$$

(9)

$$(24) (-2-i) + (3+5i) = 1+4i \quad \text{O u r I}$$

$$\begin{aligned} (25) (a) \quad & 3x^2 + 5x - 2 \\ &= 3x^2 + 6x - x - 2 \\ &= 3x(x+2) - (x+2) = \boxed{(3x-1)(x+2)} \end{aligned}$$

$$\begin{aligned} (b) \quad & 16x^4 - y^8 \\ &= (4x^2 - y^4)(4x^2 + y^4) \\ &= \boxed{(2x - y^2)(2x + y^2)(4x^2 + y^4)} \end{aligned}$$

$$\begin{aligned} (c) \quad & 2(x^2 - 5x - 14) \\ &= \boxed{2(x-7)(x+2)} \end{aligned}$$

$$\begin{aligned} (d) \quad & x^3 + 3x^2 - 4x - 12 \\ &= x^2(x+3) - 4(x+3) \\ &= (x^2 - 4)(x+3) \\ &= \boxed{(x-2)(x+2)(x+3)} \end{aligned}$$

$$\begin{aligned} (26) \quad & 2x^2 - 3x = 2 \\ & 2x^2 - 3x - 2 = 0 \\ & 2x^2 - 4x + x - 2 = 0 \\ & 2x(x-2) + 1(x-2) = 0 \\ & (2x+1)(x-2) = 0 \end{aligned}$$

$$\boxed{\begin{aligned} x &= -1/2 \\ x &= 2 \end{aligned}}$$

(10)

$$(27) 3x^2 = 6x - 15$$

$$x^2 = 2x - 5$$

$$x^2 - 2x = -5$$

$$x^2 - 2x + 1 = -5 + 1$$

$$(x-1)^2 = -4$$

$$x-1 = \pm\sqrt{4}$$

$$x-1 = \pm 2i \rightarrow x = 1 \pm 2i$$

$$(28) (a) b^2 - 4ac = (-3)^2 - 4(2)(4) = -23$$

imaginary roots

$$(b) b^2 - 4ac = (-1)^2 - 4\left(\frac{1}{3}\right)(-6) = 9$$

roots are real, rational, unequal

$$(c) b^2 - 4ac = (-6)^2 - 4(1)(9) = 0$$

roots are real, rational, equal

$$(29) 2x^2 - 6x + 10 = 0$$

$$\text{sum} = -\frac{b}{a} = \frac{6}{2} = 3 \quad \text{product} = \frac{c}{a} = \frac{10}{2} = 5$$

$$(30) 3 + \sqrt{2} + 3 - \sqrt{2} = 6$$

$$(3 + \sqrt{2})(3 - \sqrt{2}) = 9 - 2 = 7$$

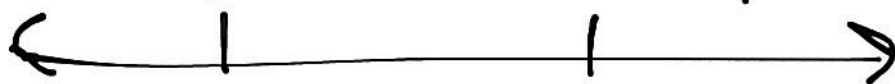
$$x^2 - \text{sum} \cdot x + \text{product} = 0$$

$$x^2 - 6x + 7 = 0$$

$$\textcircled{32} \quad x^2 - x > 6$$

$$x^2 - x - 6 > 0$$

$$(x-3)(x+2) > 0$$



Test $x = -3$	-2	Test $x = 3$	Test $x = 4$
$(-6)(-1)$ $= 6$		$(-3)(2)$ $= -6$	$(1)(6)$ $= 6$

Set builder notation:
 $\{x | x \in \mathbb{R}, x < -2 \vee x > 3\}$

$$\textcircled{31} \quad |3m - 6| + 4 \geq 22$$

$$|3m - 6| \geq 18$$

$$3m - 6 \geq 18$$

$$3m \geq 24$$

$$m \geq 8$$

$$3m - 6 \leq -18$$

$$3m \leq -12$$

$$m \leq -4$$

$$\{m | m \in \mathbb{R}, m \leq -4 \vee m \geq 8\}$$

$\textcircled{12}$

$$\begin{aligned}
 \textcircled{11a} \quad & \frac{1 + \frac{2}{a}}{\frac{2}{a} - \frac{a}{2}} \cdot \frac{2a}{2a} = \frac{2a + 4}{4 - a^2} \\
 & = \frac{2(a+2)}{(2+\cancel{a})(2-a)} \\
 & = \boxed{\frac{2}{2-a}}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{11b} \quad & \frac{x - \frac{9}{x}}{1 + \frac{3}{x}} \cdot \frac{x}{x} = \frac{x^2 - 9}{x + 3} \\
 & = \frac{(x+\cancel{3})(x-3)}{\cancel{x+3}} \\
 & = \boxed{x-3}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{11d} \quad & \frac{\frac{c}{2} - \frac{2}{c}}{1 + \frac{c}{2}} \cdot \frac{2c}{2c} = \frac{c^2 - 4}{2c + c^2} \\
 & = \frac{(c+\cancel{2})(c-2)}{c(2+\cancel{c})} \\
 & = \boxed{\frac{c-2}{c}}
 \end{aligned}$$

$$(33) \frac{1}{x} < 1$$

$$\frac{1}{x} - 1 < 0$$

$$\frac{1}{x} - \frac{x}{x} < 0$$

$$\frac{1-x}{x} < 0$$



Test
 $x = -1$

$$\frac{\frac{2}{-1}}{-1} = -2$$

Test
 $x = \frac{1}{2}$

$$\frac{\frac{\frac{1}{2}}{\frac{1}{2}}}{\frac{1}{2}} = 1$$

Test $x = 2$

$$\frac{-\frac{1}{2}}{2} = -0.5$$

Set builder notation:

$$\{x \mid x \in \mathbb{R}, x < 0 \vee x > 1\}$$

$$(34) \quad x^2 - 7x \leq x$$

$$x^2 - 8x \leq 0$$

$$x(x - 8) \leq 0$$



Test
 $x = -1$:
 $-1(-9)$
 $= 9$

Test
 $x = 1$:
 $1(-7)$
 $= -7$

Test $x = 9$:
 $9(1)$
 $= 9$

Interval notation:
 $[0, 8]$

$$(35) \quad |12 - 3x| - 2 \geq 28$$

$$|12 - 3x| \geq 30$$

$$12 - 3x \geq 30$$

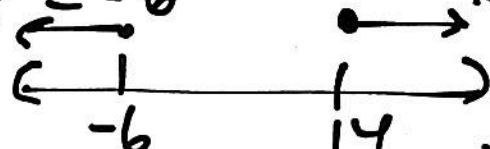
$$-3x \geq 18$$

$$x \leq -6$$

$$12 - 3x \leq -30$$

$$-3x \leq -42$$

$$x \geq 14$$



$(-\infty, -6] \cup [14, \infty)$

$$\begin{aligned}
 (3b) \quad & \frac{9}{x-4} \geq -6 \\
 & \frac{9}{x-4} + 6 \geq 0 \\
 & \frac{9}{x-4} + \frac{6(x-4)}{x-4} \geq 0 \\
 & \frac{9 + 6x - 24}{x-4} \geq 0 \\
 & \frac{6x - 15}{x-4} \geq 0
 \end{aligned}$$

Interval notation:
 $(-\infty, 2.5] \cup (4, \infty)$



<p>Test $x = 0$:</p> $\frac{-15}{-4} = 15/4$	<p>Test $x = 3$:</p> $\frac{3}{-1} = -3$	<p>Test $x = 5$:</p> $\frac{15}{1} = 15$
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$$\begin{aligned} (37) \quad & x^3 + 3x^2 - 9x - 27 = 0 \\ & x^2(x+3) - 9(x+3) = 0 \\ & (x^2 - 9)(x+3) = 0 \\ & (x-3)(x+3)(x+3) = 0 \\ & \boxed{x = 3 \quad x = -3 \quad x = -3} \end{aligned}$$

$$\begin{aligned} (38) \quad & x^4 - 8x^2 + 16 = 0 \\ & (x^2 - 4)(x^2 - 4) = 0 \\ & (x-2)(x+2)(x-2)(x+2) = 0 \\ & \boxed{x = 2 \quad x = -2 \quad x = 2 \quad x = -2} \end{aligned}$$

$$\begin{aligned} (39) \quad & (x^2 + 5x - 7)(x+3) = 0 \\ & x^2 + 5x - 7 = 0 \quad x+3 = 0 \\ & x = \frac{-5 \pm \sqrt{25 - 4(1)(-7)}}{2} \quad x = -3 \\ & \boxed{x = \frac{-5 \pm \sqrt{53}}{2}} \end{aligned}$$

40) a) Domain: $[-3, 3]$
Range: $[-1, 2]$
not a function

b) Domain: $\{0, 2, 4, 6\}$
Range: $\{5, 8, 10, 12, 20\}$
not a function

c) Domain: $[-1, 3]$
Range: $[0, 4]$
function

d) Domain: $(-5, 5)$
Range: $[-2, 2]$
function

41) $f(-2) = 4 - (-2)^2$
 $= 4 - 4 = 0$

Note: For purposes of deciding if these are onto, we are mapping from reals to reals.

(42) linear, one-to-one, onto

(43) not a function

(44) not a function

(45) constant, not one-to-one, not onto

(46) Abs. value, not one-to-one, not onto

(47) quadratic, not one-to-one, not onto

(48)

(a) $\{x | x \in \mathbb{R}, x \neq -4 \text{ or } 5\}$

(b) $\{x | x \in \mathbb{R}, x \geq 5/2\}$

(c) $\{x | x \in \mathbb{R}, x > 5\}$

$$(49) \text{ (a) } f(g(1))$$

$$g(1) = -3(1) + 1 = -2$$

$$f(-2) = 2\sqrt{-2+3} = \boxed{2}$$

$$(b) \ g(f(1))$$

$$f(1) = 2\sqrt{1+3} = 2\sqrt{4} = 4$$

$$g(4) = -3(4) + 1 = -12 + 1 = \boxed{-11}$$

$$(c) \ g(f(6))$$

$$f(6) = 2\sqrt{6+3} = 2\sqrt{9} = 6$$

$$g(6) = -3(6) + 1 = -18 + 1 = \boxed{-17}$$

$$(50) \text{ (a) } f(g(x))$$

$$= f\left(\frac{x}{x+3}\right)$$

$$= 4\left(\frac{x}{x+3}\right) + 3$$

$$\text{or } \frac{7x+9}{x+3} \text{ if simplified}$$

$$\begin{aligned}
 (50) \quad (b) \quad & g(f(x)) \\
 &= g(4x+3) \\
 &= \boxed{\frac{4x+3}{4x+3+3} \text{ or } \frac{4x+3}{4x+6}}
 \end{aligned}$$

(51) Look at graph of $y = -x^2 - 2$
 The largest y value is -2
 so the largest element of the range is -2 .

$$(52) \quad y = \frac{7-8x}{3} \quad \boxed{f(x) \text{ is } 1-1}$$

$$\text{Inv: } x = \frac{7-8y}{3}$$

$$3x = 7-8y$$

$$3x - 7 = -8y$$

$$\boxed{\frac{3x-7}{-8} = y = f^{-1}(x)}$$

(21)

⑤③ Domain : $\{1, 3, 5, 7\}$ $f(x)$ is 1-1.
Range : $\{2, 4, 6, 8\}$

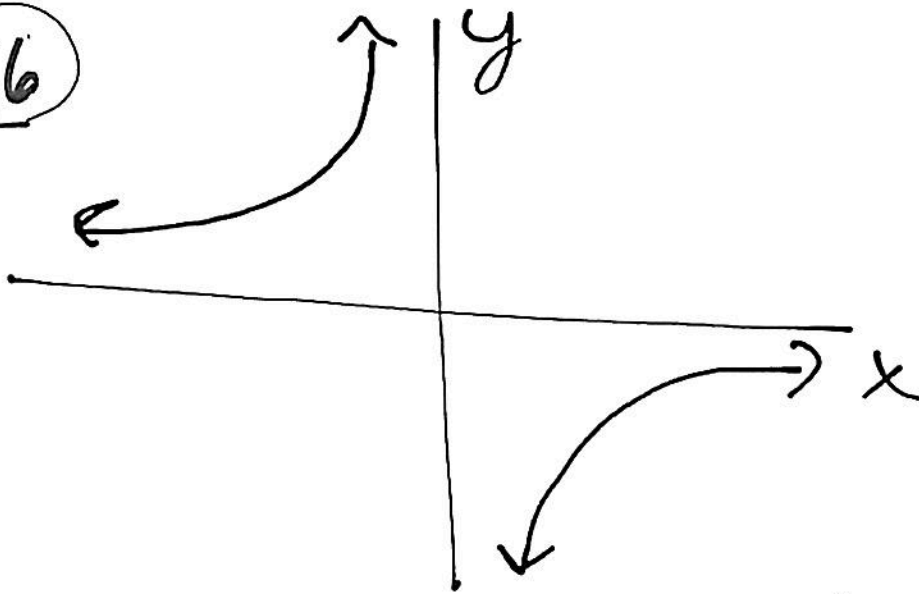
Inverse : $\{(2, 1), (4, 3), (6, 5), (8, 7)\}$

⑤④ $\frac{3}{9} = \frac{x}{4.5}$
 $9x = 13.5$
 $x = 1.5$

⑤⑤ $5(a) = 20x$
 $45 = 20x$
 $2.25 = x$

②②

56



Quad II part is made up of
points such as $(-18, 1)$, $(-9, 2)$,
 $(-6, 3)$, $(-3, 6)$, $(-2, 9)$, $(-1, 18)$

Quad IV part is made up of
points such as $(1, -18)$, $(2, -9)$,
 $(3, -6)$, $(6, -3)$, $(9, -2)$, $(18, -1)$

23

57

$$\frac{50}{5000} = \frac{80}{x}$$

$$50x = 160000$$

$$x = 3200$$

58 $6(30) = 90x$

$$180 = 90x$$

$$2 = x$$

59 $|4a + 6| - 4a = -10$

$$|4a + 6| = 4a - 10$$

$$4a + 6 = 4a - 10$$

$$6 = -10$$

no solution

$$4a + 6 = -4a + 10$$

$$8a = 4$$

$$a = \frac{1}{2}$$

reject

because does not work
in original equation

so $\boxed{\text{no solution}}$

(24)

$$(60) |4x-5| = 3$$

$$4x-5 = 3$$

$$4x = 8$$

$$x = 2$$

$$4x-5 = -3$$

$$4x = 2$$

$$x = \frac{1}{2}$$

$$(61) (a) \frac{2}{4} \cdot \frac{1}{x^2} \cdot \frac{y^5}{y^2}$$

$$= \frac{1}{2} \cdot \frac{1}{x^2} \cdot y^3$$

$$= \frac{y^3}{2x^2}$$

$$(b) 3x^{-4}y^5 \cdot (2x^3y^{-7})^2$$

$$= 3x^{-4}y^5 \cdot 4x^6y^{-14}$$

$$= 12x^2y^{-9} = \frac{12x^2}{y^9}$$

$$(62) x^{-2/5} = \frac{1}{x^{2/5}} = \frac{1}{\sqrt[5]{x^2}}$$